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# Neutrino Superbeam and Factory Tests of Grand Unified Model Predictions for the Large Mixing Angle and LOW Solar Neutrino Solutions

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Within the framework of a representative SO(10) GUT model that can accommodate both the LMA and LOW solar neutrino mixing solutions, we present explicit predictions for the neutrino oscillation parameters $\sin^2 2\theta_{13}$, $\sin^2 2\theta_{12}$, $\sin^2 2\theta_{23}$, $\Delta m^2_{21}$, and $\delta_{CP}$. The suitability of Neutrino Superbeams and Neutrino Factories for precision tests of the two model versions is discussed.

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Over the last few years the evidence for neutrino oscillations between the three known neutrino flavors ($\nu_e, \nu_\mu$, and $\nu_\tau$) has become increasingly convincing. The atmospheric neutrino flux measurements from the Super-Kamiokande (Super-K) experiment exhibit a deficit of muon neutrinos which varies with zenith angle (and hence baseline) in a way consistent with $\nu_\mu \to \nu_\tau$ oscillations [1]. In addition, recent combined evidence from Super-K and the SNO experiments [2] indicate that some electron-neutrinos from the sun are oscillating into muon and/or tau neutrinos. While the atmospheric neutrino data with $\nu_\mu \to \nu_\tau$ oscillations points to a small region of the mixing parameter space [1], the solar neutrino data is consistent with at least two regions of parameter space [3], corresponding to either the Large Mixing Angle (LMA) or to the LOW MSW [4] solution.

Neutrino oscillation data constrain Grand Unified Theories (GUTs) which provide a theory of flavor and relate lepton masses and mixings to quark masses and mixings. It is known that the presently implied neutrino mass scales can be accommodated naturally within the framework of GUTs by the seesaw mechanism [5]. In this paper we show that there exists a representative GUT model that can accommodate both the LMA and LOW solutions. We use this model to examine how Neutrino Superbeams and Neutrino Factories [6] can further test GUTs, and hence show these new facilities are necessary.

Within the framework of three-flavor mixing, the flavor eigenstates $\nu_\alpha$ ($\alpha = e, \mu, \tau$) are related to the mass eigenstates $\nu_j$ ($j = 1, 2, 3$) in vacuum by

$$\nu_\alpha = \sum_j U_{\alpha j}\nu_j \ , \quad U \equiv U_{MNS}\Phi_M \tag{1}$$

where $U$ is the unitary $3\times 3$ Maki-Nakagawa-Sakata (MNS) mixing matrix [7] times a diagonal phase matrix $\Phi_M = \mathrm{diag}(e^{i\chi_1},\ e^{i\chi_2},\ 1)$. The MNS matrix is conventionally parametrized by 3 mixing angles $(\theta_{23}, \theta_{12}, \theta_{13})$ and a CP-violating phase, $\delta_{CP}$:

$$U_{MNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}\xi^* \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}\xi & c_{12}c_{23} - s_{12}s_{23}s_{13}\xi & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}\xi & -c_{12}s_{23} - s_{12}c_{23}s_{13}\xi & c_{23}c_{13} \end{pmatrix} \tag{2}$$

where $c_{jk} \equiv \cos\theta_{jk}$, $s_{jk} \equiv \sin\theta_{jk}$ and $\xi = e^{i\delta_{CP}}$. The three angles can be restricted to the first quadrant, $0 \leq \theta_{ij} \leq \pi/2$, with $\delta_{CP}$ in the range $-\pi \leq \delta_{CP} \leq \pi$, though it will later prove advantageous to consider $\theta_{13}$ in the fourth quadrant for the LMA solutions.

The atmospheric neutrino oscillation data indicate that [1]

$$\begin{aligned} \Delta m^2_{32} &\simeq 3.0\times 10^{-3}\ \mathrm{eV}^2, \\ \sin^2 2\theta_{23} &= 1.0,\ (\geq 0.89 \text{ at } 90\%\text{ c.l.}), \end{aligned} \tag{3}$$

where $\Delta m^2_{ij} \equiv m^2_i - m^2_j$ and $m_1$, $m_2$ and $m_3$ are the mass eigenstates. The atmospheric neutrino oscillation amplitude can be expressed solely in terms of the $U_{MNS}$ matrix elements and is given by $\sin^2 2\theta_{atm} = 4|U_{\mu 3}|^2(1 - |U_{\mu 3}|^2) \simeq 4|U_{\mu 3}|^2|U_{\tau 3}|^2$. The approximation is valid because $|U_{e3}|$ is known to be small [8].

The solar neutrino oscillation data from Super-K indicate that, for the LMA solution, the allowed region is approximately bounded by

$$\begin{aligned} \Delta m^2_{21} &\simeq (2.2 - 17)\times 10^{-5}\ \mathrm{eV}^2, \\ \sin^2 2\theta_{sol} &\simeq (0.6 - 0.9), \end{aligned} \tag{4}$$

while for the LOW solution,

$$\begin{aligned} \Delta m^2_{21} &\simeq (0.3 - 2)\times 10^{-7}\ \mathrm{eV}^2, \\ \tan^2\theta_{12} &\simeq (0.6 - 1.2), \end{aligned} \tag{5}$$

where the solar neutrino oscillation amplitude is $\sin^2 2\theta_{sol} = 4|U_{e1}|^2(1 - |U_{e1}|^2) \simeq 4|U_{e1}|^2|U_{e2}|^2$, while $\tan^2\theta_{12} = |U_{e2}/U_{e1}|^2$.

The GUT model we consider was developed by Albright and Barr [9] and is based on an $SO(10)$ GUT with a $U(1)\times Z_2\times Z_2$ flavor symmetry. The model involves a minimum set of Higgs fields which solves the doublet-triplet splitting problem. The Higgs superpotential exhibits the $U(1)\times Z_2\times Z_2$ symmetry which is used for

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the flavor symmetry of the GUT model. Details of the model can be found in [9]. We note that the Dirac mass matrices $U,\ D,\ N,\ L$ for the up quarks, down quarks, neutrinos and charged leptons, respectively, are found for $\tan\beta \simeq 5$ to be

$$U = \begin{pmatrix} \eta & 0 & 0 \\ 0 & 0 & \epsilon/3 \\ 0 & -\epsilon/3 & 1 \end{pmatrix},\ D = \begin{pmatrix} 0 & \delta & \delta' e^{i\phi} \\ \delta & 0 & \sigma+\epsilon/3 \\ \delta' e^{i\phi} & -\epsilon/3 & 1 \end{pmatrix}$$
$$N = \begin{pmatrix} \eta & 0 & 0 \\ 0 & 0 & -\epsilon \\ 0 & \epsilon & 1 \end{pmatrix},\ L = \begin{pmatrix} 0 & \delta & \delta' e^{i\phi} \\ \delta & 0 & -\epsilon \\ \delta' e^{i\phi} & \sigma+\epsilon & 1 \end{pmatrix}, \tag{6}$$

where $U$ and $N$ are scaled by $M_U$, and $D$ and $L$ are scaled by $M_D$. All nine quark and charged lepton masses, plus the three CKM angles and CP phase, are well-fitted with the eight input parameters

$$\begin{array}{llll} M_U & \simeq 113 \text{ GeV}, & M_D & \simeq 1 \text{ GeV}, \\ \sigma & = 1.78, & \epsilon & = 0.145, \\ \delta & = 0.0086, & \delta' & = 0.0079, \\ \phi & = 126^\circ, & \eta & = 8\times 10^{-6}, \end{array} \tag{7}$$

defined at the GUT scale to fit the low scale observables after evolution downward from $\Lambda_{GUT}$:

$$\begin{array}{llll} m_t(m_t) & = 165 \text{ GeV}, & m_\tau & = 1.777 \text{ GeV}, \\ m_u(1 \text{ GeV}) & = 4.5 \text{ MeV}, & m_\mu & = 105.7 \text{ MeV}, \\ V_{us} & = 0.220, & m_e & = 0.511 \text{ MeV}, \\ V_{cb} & = 0.0395, & \delta_{CP} & = 64^\circ. \end{array} \tag{8}$$

These lead to the following predictions:

$$\begin{array}{ll} m_b(m_b) = 4.25 \text{ GeV}, & m_c(m_c) = 1.23 \text{ GeV}, \\ m_s(1 \text{ GeV}) = 148 \text{ MeV}, & m_d(1 \text{ MeV}) = 7.9 \text{ MeV}, \\ |V_{ub}/V_{cb}| = 0.080, & \sin 2\beta = 0.64. \end{array} \tag{9}$$

With no extra phases present, the vertex of the CKM unitary triangle occurs near the center of the presently allowed region with $\sin 2\beta \simeq 0.64$, comparing favorably with recent results [10]. The Hermitian matrices $U^\dagger U,\ D^\dagger D$, and $N^\dagger N$ are diagonalized with small left-handed rotations, $U_U,\ U_D,\ U_N$, respectively, while $L^\dagger L$ is diagonalized by a large left-handed rotation, $U_L$. This accounts for the small value of $|V_{cb}| = |(U_U^\dagger U_D)_{cb}|$, while $|U_{\mu 3}| = |(U_L^\dagger U_\nu)_{\mu 3}|$ will turn out to be large for any reasonable right-handed Majorana mass matrix, $M_R$ [11].

The effective light neutrino mass matrix, $M_\nu$, is obtained from the seesaw mechanism once $M_R$, is specified. While the large atmospheric neutrino mixing $\nu_\mu \leftrightarrow \nu_\tau$ arises primarily from the structure of the charged lepton mass matrix, the structure of $M_R$ determines the type of $\nu_e \leftrightarrow \nu_\mu,\ \nu_\tau$ solar neutrino mixing.

To obtain the LMA solution requires some fine-tuning and a hierarchical structure for $M_R$, but this can be explained in terms of Froggatt-Nielsen diagrams [12]. Here we restrict our attention to a slightly less general form for $M_R$ than that considered in [9] and [13]:

$$M_R = \begin{pmatrix} b^2\eta^2 & -b\epsilon\eta & a\eta \\ -b\epsilon\eta & \epsilon^2 & -\epsilon \\ a\eta & -\epsilon & 1 \end{pmatrix} \Lambda_R, \tag{10}$$

where the parameters $\epsilon$ and $\eta$ are those introduced in Eq.(6) for the Dirac sector. This structure for $M_R$ can be understood as arising from one Higgs singlet which induces a $\Delta L = 2$ transition and contributes to all nine matrix elements while, by virtue of its flavor charge assignment, a second Higgs singlet breaks lepton number but modifies only the 13 and 31 elements of $M_R$. As shown in detail in [13], we can introduce additional CP violation by assigning a relative phase to the two lepton number breaking Higgs singlets, whereby we set

$$a = b - a' e^{i\phi'}. \tag{11}$$

The LOW solution, on the other hand, can be obtained with a simple hierarchical structure for $M_R$ of the form

$$M_R = \begin{pmatrix} e & d & 0 \\ d & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \Lambda_R, \tag{12}$$

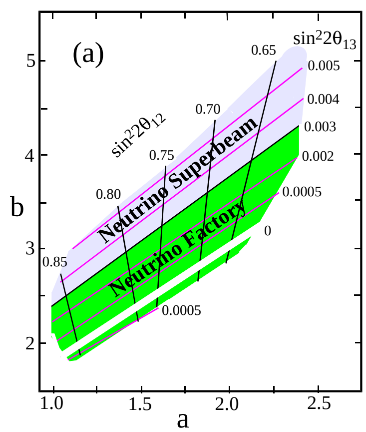
where by the flavor charge assignments, one Higgs singlet inducing a $\Delta L = 2$ transition contributes to the 12, 21 and 33 elements, while a second Higgs singlet also breaks lepton number but contributes only to the 11 matrix element. For simplicity we keep both $d$ and $e$ real, since the leptonic CP phase is inaccessible to measurement for $\Delta m^2_{21}$ values in the LOW region.

For either the LMA or LOW version, $M_\nu$ is then obtained by the seesaw formula [5], $M_\nu = N^T M_R^{-1} N$. With $M_\nu$ complex symmetric, both $M_\nu^\dagger M_\nu$ and $M_\nu$ itself can be diagonalized by the same unitary transformation, $U_\nu$, where in the latter case we find

$$U_\nu^T M_\nu U_\nu = \text{diag}(m_1,\ -m_2,\ m_3). \tag{13}$$

With real light neutrino masses, $U_\nu$ can not be arbitrarily phase transformed and is uniquely specified up to sign changes on its column eigenvectors [13]. Hence $U_{MNS}$ is found by applying arbitrary phase transformations on $U_L^\dagger U_\nu$ to bring that into the parametric form of Eq. (2) whereby the $e1,\ e2,\ \mu 3$ and $\tau 3$ elements are real and positive, the real parts of the $\mu 2$ and $\tau 1$ elements are positive, while the real parts of the $\mu 1$ and $\tau 2$ elements are negative. The inverse phase transformation of that applied on the right can then be identified with the Majorana phase matrix, $\Phi_M$ of Eq. (2). The evolution of the predicted values between the GUT scale and the low scales can be safely ignored [14], since $\tan\beta \simeq 5$ is moderately low and the neutrino mass spectrum is hierarchical with the opposite CP parities present in Eq. (13).

We can now examine the viable region of GUT model parameter space that is consistent with either the LMA or LOW solar neutrino solution, and explore the predicted relationships among the observables



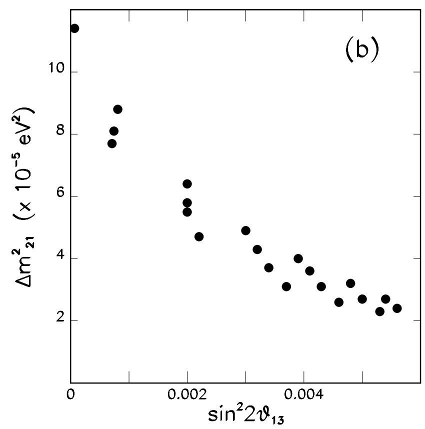




FIG. 1: (a) The viable region of GUT parameter space consistent with the present bounds on the LMA MSW solution. Contours of constant $\sin^2 2\theta_{13}$ and lines of constant $\sin^2 2\theta_{12}$ are shown. (b) Variation of $\sin^2 2\theta_{13}$ with $\Delta m^2_{21}$. The points plotted populate a grid which spans the viable region of the $(a, b)$ parameter space.

TABLE I: List of four points selected in the LMA allowed parameter region to illustrate the neutrino oscillation parameter predictions of the GUT model. The CP phase $\delta_{CP}$ arises from $\phi$ in $L$ alone, as no phase has been introduced in $M_R$.

| $a$ | $b$ | $\Delta m^2_{21}$ $(eV^2)$ | $\Delta m^2_{32}$ $(eV^2)$ | $\tan^2\theta_{12}$ | $\sin^2 2\theta_{12}$ | $\sin^2 2\theta_{23}$ | $\sin^2 2\theta_{13}$ | $\delta_{CP}$ |
|---|---|---|---|---|---|---|---|---|
| 1.0 | 2.0 | $6.5\times10^{-5}$ | $3.2\times10^{-3}$ | 0.49 | 0.88 | 0.994 | 0.0008 | $-4°$ |
| 1.7 | 2.7 | $10.9\times10^{-5}$ | $3.2\times10^{-3}$ | 0.32 | 0.73 | 0.996 | 0.00008 | $-14°$ |
| 1.7 | 3.4 | $4.0\times10^{-5}$ | $3.2\times10^{-3}$ | 0.33 | 0.75 | 0.992 | 0.0033 | $-2°$ |
| 2.2 | 3.5 | $8.8\times10^{-5}$ | $3.2\times10^{-3}$ | 0.24 | 0.63 | 0.996 | 0.0008 | $-4°$ |

$\sin^2 2\theta_{23}, \sin^2 2\theta_{12}, \sin^2 2\theta_{13}, \delta_{CP}$, $\Delta m^2_{32}$, and $\Delta m^2_{21}$. We shall emphasize here the simpler cases in which there are, in effect, only two additional real dimensionless GUT model parameters, $a$ and $b$ in the LMA version or $d$ and $e$ in the LOW version. In either version, the third parameter $\Lambda_R$ sets the scale of $\Delta m^2_{32}$.

The viable region of GUT model parameter space consistent with the LMA solar solution is shown in Fig. 1(a). Both parameters $a$ and $b$ are constrained by the data to be close to unity, with $1.0 \lesssim a \lesssim 2.4$ and $1.8 \lesssim b \lesssim 5.2$. Superimposed on the allowed region, Fig. 1 shows contours of constant $\sin^2 2\theta_{12}$ and contours of constant $\sin^2 2\theta_{13}$. The region above $\sin^2 2\theta_{13} = 0.003$ can be explored with Neutrino Superbeams, while the region below this can be explored with Neutrino Factories, down to $\sin^2 2\theta_{13} \sim 0.0001$. Figure 1(b) displays an approximate correlation between the predicted values of $\Delta m^2_{21}$ and $\sin^2 2\theta_{13}$. The points are confined to a narrow band, with $\sin^2 2\theta_{12}$ varying across the band. Note that if the LMA solution is indeed the correct solution, KamLAND [15] is expected to provide measurements of $\Delta m^2_{21}$ and $\sin^2 2\theta_{12}$. Hence the GUT model we are considering will be able to give a precise prediction for $\sin^2 2\theta_{13}$. In Table I we have selected four points in the LMA allowed parameter region to illustrate the neutrino oscillation predictions of the GUT model. The correlations noted above are evident. It is also striking how nearly maximal are the values for the atmospheric mixing parameter, $\sin^2 2\theta_{23}$. However, if an additional phase is incorporated into $M_R$ for this LMA case as indicated in Eq. (11), the maximality of the atmospheric solution is decreased to the lower bound in Eq. (4) as $|\delta_{CP}|$ approaches 50°. See [13] for more details.

Turning now to the GUT model version for the LOW solution, we find that there are two parametric regions shown in Fig. 2 for the presently allowed solutions, corresponding to $-5.0 \lesssim d\times10^5 \lesssim -2.4$, $0 \lesssim e\times10^9 \lesssim 13$ and $3.0 \lesssim d\times10^5 \lesssim 6.0$, $3 \lesssim e\times10^9 \lesssim 24$. Here no dramatic correlation between $\sin^2 2\theta_{13}$ and $\Delta m^2_{21}$ exists. But if Borexino, for example, determines $\sin^2 2\theta_{12}$ and $\Delta m^2_{21}$ with nearly 1% precision, $\sin^2 2\theta_{13}$ will be specified up to a two-fold ambiguity in the GUT model in question. A first measurement of $\sin^2 2\theta_{13}$ would resolve the ambiguity, and a precise measurement would test the model. For the negative $d$ version, a SuperBeam facility will be able to test the model, while for the positive $d$ version and most of the allowed region, the model can be tested only with a Neutrino Factory. Table II gives the relevant mixing solutions for another set of four points. In contrast to the LMA results with small CP phases, we see that the atmospheric mixing is not nearly so maximal.

In conclusion, we have studied predictions for a particular but representative GUT model that can accommodate both the LMA and LOW solar neutrino solutions

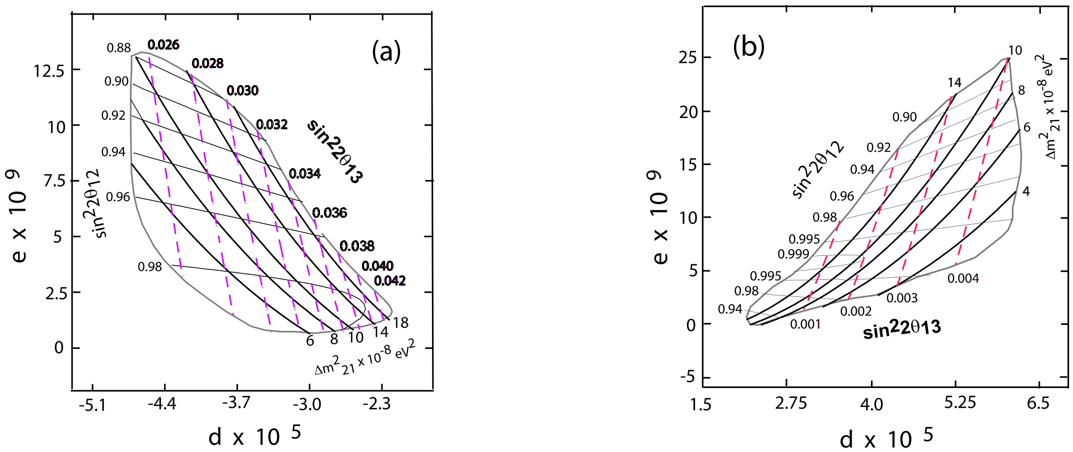



FIG. 2: The viable region of GUT parameter space consistent with the present bounds on the LOW MSW solution for (a) negative $d$ and (b) positive $d$. Contours of constant $\sin^2 2\theta_{13}$, $\sin^2 2\theta_{12}$, and $\Delta m^2_{21}$ are shown.

TABLE II: List of four points selected in the LOW allowed parameter region to illustrate the neutrino oscillation parameter predictions of the GUT model.

| $d$ | $e$ | $\Delta m^2_{21}$ $(eV^2)$ | $\Delta m^2_{32}$ $(eV^2)$ | $\tan^2\theta_{12}$ | $\sin^2 2\theta_{12}$ | $\sin^2 2\theta_{23}$ | $\sin^2 2\theta_{13}$ |
|---|---|---|---|---|---|---|---|
| $-4.2\times10^{-5}$ | $10.0\times10^{-9}$ | $1.20\times10^{-7}$ | $3.0\times10^{-3}$ | 0.56 | 0.906 | 0.911 | 0.028 |
| $-3.6\times10^{-5}$ | $3.0\times10^{-9}$ | $0.64\times10^{-7}$ | $3.0\times10^{-3}$ | 0.86 | 0.980 | 0.898 | 0.030 |
| $3.6\times10^{-5}$ | $5.0\times10^{-9}$ | $0.98\times10^{-7}$ | $3.0\times10^{-3}$ | 1.00 | 0.999 | 0.914 | 0.0016 |
| $5.0\times10^{-5}$ | $13.0\times10^{-9}$ | $0.85\times10^{-7}$ | $3.0\times10^{-3}$ | 0.70 | 0.966 | 0.918 | 0.0033 |

and find that precise measurements of $\sin^2 2\theta_{12}$, $\Delta m^2_{21}$, and $\sin^2 2\theta_{13}$ are needed to test the theory. The LMA solution, which requires some fine tuning of the $M_R$ mass matrix, also requires $\sin^2 2\theta_{13} \lesssim 0.006$. For the LOW solution which requires no fine tuning, $\sin^2 2\theta_{13}$ can be as small or an order of magnitude larger depending upon the sign of the $d$ model parameter in $M_R$. Our work suggests progress on testing GUTs can be made with Neutrino Superbeams, but ultimately a Neutrino Factory will be needed to help identify the correct model.

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